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FRACTIONAL RESOLVENT OPERATOR WITH $\alpha \in (0,1)$ AND APPLICATIONS

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Abstract. In this paper we study an analytic resolvent family for abstract fractional integro-differential system using the perturbation theory of sectorial operators. We apply this resolvent family on the existence of mild solutions for abstract semilinear Cauchy problem

$$\begin{aligned} D_t^\alpha u(t) &= Au(t) + \int_0^t B(t-s)u(s)ds + f(t,u(t)), t \in (0,\tau), \\ u(0) &= u_0 \in X, \end{aligned}$$

where $D_t^\alpha u$ represents the Caputo derivative of u for $\alpha \in (0,1)$, $A, (B(t))_{t\geqslant 0}$ are closed linear operators defined on a common domain which is dense in a Banach space X and f satisfies appropriated conditions. In the end, we applain the ours abstract results in the existence of mild solution of two partial integro-differential systems.

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