

## UNIQUE POSITIVE SOLUTION FOR NONLINEAR CAPUTO-TYPE FRACTIONAL $q$ -DIFFERENCE EQUATIONS WITH NONLOCAL AND STIELTJES INTEGRAL BOUNDARY CONDITIONS

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*Abstract.* This paper contains a new discussion for the type of generalized nonlinear Caputo fractional  $q$ -difference equations with  $m$ -point boundary value problem and Riemann-Stieltjes integral  $\tilde{\alpha}[x] := \int_0^1 x(t)d\Lambda(t)$ . By applying the fixed point theorem in cones, we investigate an existence of a unique positive solution depends on  $\lambda > 0$ . We present some useful properties related to the Green's function for  $m$ -point boundary value problem.

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