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BLOW UP OF NONAUTONOMOUS FRACTIONAL REACTION–DIFFUSION SYSTEMS

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Abstract. We provide a sufficient condition for finite time blow up of the positive mild solution to the nonautonomous Cauchy problem of a reaction-diffusion system with distinct fractional diffusions. The proof is based on the reduction to an ordinary differential system by means of a comparison between the transition densities of the semigroups generated by the different fractional Laplacians. Moreover, we prove that this condition is also a sufficient condition for the blow up of a related nonautonomous fractional diffusion-convection-reaction system.

Mathematics subject classification (2010): 35B44, 35C15, 35K57, 35S10.
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REFERENCES

[1] J. BEBERNES AND D. EBERLY, *Mathematical problems from combustion theory*, Applied Mathematical Sciences **83**, Springer-Verlag, New York, 1989.
[2] K. BOGDAN AND T. JAKUBOWSKI, *Estimates of heat kernel of fractional Laplacian perturbed by gradient operators*, Commun. Math. Phys. **271**, (2007), 179–198.
[3] M. ESCOBEDO AND M. A. HERRERO, *Boundedness and blow up for a semilinear reaction-diffusion system*, J. Differential Equations **89**, (1991), 176–202.
[4] A. Z. FINO AND M. KIRANE, *Qualitative properties of solutions to a time-space fractional evolution equation*, Quart. Appl. Math. **70**, 1 (2012), 133–157.
[5] H. FUJITA, *On the blowing up of solutions of the Cauchy problem for* $u_t = \Delta u + u^{1+\alpha}$, J. Fac. Sci. Univ. Tokyo Sect. I **13**, (1966), 109–124.
[6] M. GUEDDA AND M. KIRANE, *Criticality for some evolution equations*, Differential Equations **37**, 4 (2001), 511–520.
[7] T. KAKEHI AND Y. OSHITA, *Blowup and global existence of a solution to a semilinear reaction-diffusion system with the fractional Laplacian*, Math. J. Okayama Univ. **59**, (2017), 175–218.
[8] M. KIRANE AND M. QAFSAOUI, *Global nonexistence for the Cauchy problem of some nonlinear reaction-diffusion-systems*, J. Math. Anal. Appl. **268**, (2002), 217–243.
[9] C. M. KIRK AND W. E. OLMSTEAD, *Superdiffusive blow-up with advection*, Int. J. Dynamical Systems and Differential Equations **4**, Nos. 1/2 (2012), 93–102.
[10] J. A. LÓPEZ-MIMBELA AND J. VILLA-MORALES, *Local time and Tanaka formula for a multitype Dawson-Watanabe superprocess*, Math. Nachr. **279**, 15 (2006), 1695–1708.
[11] A. PÉREZ, *A blow up condition for a nonautonomous semilinear system*, Elec. J. Diff. Equations **2006**, 94 (2006), 1–8.
[12] A. PÉREZ, *Blow up of fractional reaction-diffusion systems with and without convection terms*, J. Integral Equations Applications **30**, 1 (2018), 181–196.
[13] A. PÉREZ AND J. VILLA, *Blow-up for a system with time-dependent generators*, Lat. Am. J. Probab. Math. Stat. **7**, (2010), 207–215.
[14] A. A. SAMARSKII, V. A. GALAKTIONOV, S. P. KURDYUMOV AND A. P. MIKHAILOV, *Blow-up in quasilinear parabolic equations*, de Gruyter Expositions in Mathematics **19**, de Gruyter, Berlin, 1995.
[15] K. SATO, *Lévy processes and infinitely divisible distributions*, Cambridge Studies in Advanced Mathematics **68**, Cambridge University Press, Cambridge, 1999.

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[16] M. F. SHLESINGER, G. M. ZASLAVSKY AND U. FRISCH (EDS), *Lévy fligths and related topics in Physics*, Lecture Notes in Physics **450**, Springer-Verlag, Berlin, 1995.
[17] S. SUGITANI, *Nonexistence of global solutions for some nonlinear integral equations*, Osaka J. Math. **12**, (1975), 45–51.
[18] A. S. TERSENOV, *The preventive effect of the convection and of the diffusion in the blow-up phenomenon for parabolic equations*, Ann. I. H. Poincaré-AN **21**, (2004), 533–541.
[19] Y. UDA, *The critical exponent for a weakly coupled system of the generalized Fujita type reaction-diffusion equations*, Z. Angew. Math. Phys. **46**, (1995), 366–383.
[20] J. VILLA-MORALES, *Blow up of mild solutions of a system of partial differential equations with distinct fractional diffusions*, Elec. J. Diff. Equations **2014**, 41 (2014), 1–9.
[21] E. WU AND Y. TANG, *Blow-up solutions to the Cauchy problem of a fractional reaction-diffusion system*, J. Inequal. Appl. **2015**, 123 (2015), 1–18.