

CONTROLLABILITY AND OBSERVABILITY OF TIME-INVARIANT LINEAR NABLA FRACTIONAL SYSTEMS

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Abstract. In this paper, we study linear time-invariant nabla fractional discrete control systems. The nabla fractional difference operator is considered in the sense of Riemann-Liouville definition of the fractional derivative. We first give necessary and sufficient rank conditions for controllability of the discrete fractional system via Gramian and controllability matrices. We then obtain rank conditions for observability of the discrete fractional system. We illustrate main results with some numerical examples. We close the paper by stating that the rank conditions for the time-invariant linear dynamic system on time scales, fractional system in continuous time, and fractional system in discrete time coincide.

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