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FRACTIONAL INEQUALITIES FOR EXPONENTIALLY GENERALIZED (m,ω,h_1,h_2)–PREINVEX FUNCTIONS WITH APPLICATIONS

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Abstract. The aim of this paper is to introduce a new extension of preinvexity called exponentially generalized (m,ω,h_1,h_2)–preinvexity. Some new integral inequalities of Hermite–Hadamard type for exponentially generalized (m,ω,h_1,h_2)–preinvex functions via Riemann–Liouville fractional integral are established. We show that the class of exponentially generalized (m,ω,h_1,h_2)–preinvex functions includes several other classes of preinvex functions. At the end, some new error estimates for trapezoidal quadrature formula are provided as well. This results may stimulate further research in different areas of pure and applied sciences.

Mathematics subject classification (2010): 26D07, 26A51, 26A33, 26D10, 26D15, 90C23.
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