STUDY OF A DISCRETIZED FRACTIONAL–ORDER
ECO–EPIDEMIOLOGICAL MODEL WITH PREY INFECTION

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Abstract. In this paper, an attempt is made to understand the dynamics of a three-dimensional discrete fractional-order eco-epidemiological model with Holling type II functional response. We first discretize a fractional-order predator-prey-parasite system with piecewise constant arguments and then explore the system dynamics. Analytical conditions for the local stability of different fixed points have been determined using the Jury criterion. Several examples are given to substantiate the analytical results. Our analysis shows that stability of the discrete fractional order system strongly depends on the step-size and the fractional order. More specifically, the critical value of the step-size, where the switching of stability occurs, decreases as the order of the fractional derivative decreases. Simulation results explore that the discrete fractional-order system may also exhibit complex dynamics, like chaos, for higher step-size.

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