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SUCCESSIVE APPROXIMATIONS OF SOLUTIONS TO THE CAPUTO FRACTIONAL DIFFERENTIAL EQUATIONS

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Abstract. We consider an initial value problem involving a single term Caputo differential equation of fractional order strictly greater than one. For those with right hand sides that satisfy an Osgood type condition, we show that there exist successive approximations which converge to the solution at an exponential rate. As an application of this result, we study the Ulam-Hyers stability of these problems.

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