

MULTIPLE SOLUTIONS FOR FRACTIONAL HAMILTONIAN SYSTEMS LOCALLY DEFINED NEAR THE ORIGIN

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Abstract. In this article, we are interested in the existence of infinitely many solutions for a class of fractional Hamiltonian systems

$$\begin{cases} {}_t D_{-\infty}^{\alpha}(-{}_{\infty} D_t^{\alpha} u)(t) + L(t)u(t) = \nabla W(t, u(t)), t \in \mathbb{R} \\ u \in H^{\alpha}(\mathbb{R}), \end{cases} \quad (0.1)$$

where $L(t)$ is neither uniformly positive definite nor coercive, and $W(t, x)$ is locally defined and subquadratic or superquadratic near the origin with respect to x . The proof is based on variational methods and critical point theory.

Mathematics subject classification (2010): 34C37, 35A15, 35B38.

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