ON FRACTIONAL MEAN VALUE THEOREMS ASSOCIATED WITH HADAMARD FRACTIONAL CALCULUS AND APPLICATION

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Abstract. This paper is mainly to establish generalized mean value theorems involved with left and right Hadamard fractional calculus. In light of suitable absolutely continuous spaces and auxiliary scaling function, the novel Taylor type mean value theorem and Cauchy type mean value theorem are demonstrated in the functional space generated by logarithmic basis, respectively. Additionally, several indispensable examples are given to verify the effectiveness of our theoretical results.

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