POSITIVE SOLUTIONS FOR SEMIPOSITONE SINGULAR α–ORDER $(2 < \alpha < 3)$ FRACTIONAL BVPS ON THE HALF–LINE WITH D^β–DERIVATIVE DEPENDENCE

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Abstract. This article deals with existence of positive solutions to the fractional boundary value problem

$$\begin{cases} D^\alpha u(t) + f(t, u(t), D^\beta u(t)) = 0, \ t > 0 \\ u(0) = D^{\alpha-2} u(0) = \lim_{t \to \infty} D^{\alpha-1} u(t) = 0 \end{cases}$$

where $\alpha \in (2,3)$, $\beta \in (0, \alpha-2]$, D^α is the standard Riemann-Liouville fractional derivative and the function $f : (0, +\infty)^3 \to \mathbb{R}$ is continuous semipositone and may exhibit singular at $u = 0$ and at $D^\beta u = 0$ The main existence result is obtained by means of Guo-Krasnoselskii's version of expansion and compression of a cone principal in a Banach space.

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