

## INITIAL–BOUNDARY VALUE AND INVERSE PROBLEMS FOR SUBDIFFUSION EQUATIONS IN $\mathbb{R}^N$

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*Abstract.* An initial-boundary value problem for a subdiffusion equation with an elliptic operator  $A(D)$  in  $\mathbb{R}^N$  is considered. The existence and uniqueness theorems for a solution of this problem are proved by the Fourier method. Considering the order of the Caputo time-fractional derivative as an unknown parameter, the corresponding inverse problem of determining this order is studied. It is proved, that the Fourier transform of the solution  $\hat{u}(\xi, t)$  at a fixed time instance recovers uniquely the unknown parameter. Further, a similar initial-boundary value problem is investigated in the case when operator  $A(D)$  is replaced by its power  $A^\sigma$ . Finally, the existence and uniqueness theorems for a solution of the inverse problem of determining both the orders of fractional derivatives with respect to time and the degree  $\sigma$  are proved. We also note that when solving the inverse problems, a decrease in the parameter  $\rho$  of the Mittag-Leffler functions  $E_\rho$  has been proved.

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