

## ON NONLINEAR VOLTERRA–FREDHOLM TYPE DISCRETE FRACTIONAL SUM INEQUALITIES

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*Abstract.* In the current paper, we establish some new forms of the discrete fractional sum inequalities of the Volterra-Fredholm type. These inequalities can serve as a powerful tool in the analysis of boundedness and uniqueness of the solutions of certain Volterra-Fredholm fractional sum-difference equations and their variants. Some applications are also given to convey the importance of our results.

*Mathematics subject classification (2020):* 26D15, 39A12, 26A33.

*Keywords and phrases:* Volterra-Fredholm type, discrete fractional sum inequality, fractional sum difference equation, boundedness.

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