INTEGRAL INEQUALITIES WITHIN THE FRAMEWORK
OF GENERALIZED FRACTIONAL INTEGRALS

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Abstract. In this work, a new generalized fractional integral is defined and studied, and different relationships (equalities and inequalities) are obtained, which have as particular cases several of those reported in the literature. Hermite-Hadamard type inequalities are obtained for different kinds of functions such as symmetric, convex symmetric, Wright-quasi-convex and h-symmetrized convex.

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