Fractional Differential Calculus
Volume 11, Number 1 (2021), 85–110
doi:10.7153/fdc-2021-11-06

A UNIQUENESS CRITERION FOR NONTRIVIAL SOLUTIONS OF THE NONLINEAR HIGHER–ORDER ∇–DIFFERENCE SYSTEMS OF FRACTIONAL–ORDER

YOUSEF GHOLAMI

Abstract. The main aim of this article is to establish a uniqueness criterion for coupled systems of the nonlinear higher-order ∇-difference boundary value problems. To this end, the coincidence degree theory has chosen to make a solvability space for the existence of at least one solution to the under investigation fractional-order system. Next, we create some conditions that enable us to prove the existence of the exactly one solution of the under study fractional-order system. At the end, a numerical example is given to illustrate the applicability of the obtained theoretical criterion.

Mathematics subject classification (2020): Primary: 34A08, 34F15, 39A12; Secondary: 39A10, 34A12.
Keywords and phrases: Fractional sums and differences, coincidence degree theory, existence, uniqueness, nontrivial solution.

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