Fractional Differential Calculus
Volume 11, Number 1 (2021), 111–120
doi:10.7153/fdc-2021-11-07

A NOTE ON GENERALIZED FRACTIONAL DIFFUSION EQUATIONS ON POINCARÉ HALF PLANE

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Abstract. In this paper we study generalized time-fractional diffusion equations on the Poincaré half plane $\mathbb{H}_2^+$. The time-fractional operators here considered are fractional derivatives of a function with respect to another function, that can be obtained essentially by means of a deterministic change of variable in the Caputo derivative. We obtain an explicit representation of the fundamental solution of the generalized-diffusion equation on $\mathbb{H}_2^+$ and provide a probabilistic interpretation in terms of a time-changed hyperbolic Brownian motion. We finally include an explicit result regarding the non-linear case admitting a separating variable solution.

Mathematics subject classification (2020): 35R11, 33E12, 34A08.
Keywords and phrases: Generalized time-fractional diffusion equation, hyperbolic geometry, hyperbolic Brownian motion.

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Paper FDC-11-07