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ON THE BACKWARD PROBLEMS IN TIME FOR TIME–FRACTIONAL SUBDIFFUSION EQUATIONS

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Abstract. The backward problem for subdiffusion equation with the fractional Riemann-Liouville time-derivative of order $\rho \in (0,1)$ and an arbitrary positive self-adjoint operator A is considered. This problem is ill-posed in the sense of Hadamard due to the lack of stability of the solution. Nevertheless, we will show that if we consider sufficiently smooth current information, then the solution exists and it is unique. Using this result, we study the inverse problem of initial value identification for subdiffusion equation. The results obtained differ significantly from the corresponding results for the classical diffusion equation (i.e. $\rho = 1$) and even for the subdiffusion equation with the Caputo derivative. A list of examples of operator A is discussed, including linear systems of fractional differential equations, differential models with involution, fractional Sturm-Liouville operators, and many others.

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