

QUASI-BOUNDARY METHOD FOR A FRACTIONAL ILL-POSED PROBLEM

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Abstract. A quasi-boundary method is used to study an ill-posed, time-fractional diffusion equation involving the fractional Riemann-Liouville derivative. In particular, we consider an ill-posed problem for a family of well-posed problems, and prove, by means of eigenfunction expansions, that the solutions of the latter problems converge to the solutions associated with the former problem. The analysis presented includes providing conditions for the rate of the convergence.

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