

IMPULSIVE NABLA FRACTIONAL DIFFERENCE EQUATIONS

JAGAN MOHAN JONNALAGADDA

Abstract. This article deals with the study of impulsive nabla fractional difference equations. First, we present a first-order initial value problem (IVP) on impulsive nabla difference equations and write its equivalent sum equation. To illustrate the proposed procedure, we provide an example. In this line, we propose a particular class of IVPs for α^{th} -order ($0 < \alpha \leq 1$) impulsive nabla difference equations in the Caputo sense and establish its equivalent sum equation. We furnish an eigenvalue problem to demonstrate the proposed method. Next, we introduce a special class of α^{th} -order ($0 < \alpha \leq 1$) impulsive nabla boundary value problems (BVPs) and analyse its solutions, using fixed point theorems. Finally, we support this analysis through a few examples.

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