

## A NEW GENERALIZATION OF $q$ -HERMITE-HADAMARD TYPE INTEGRAL INEQUALITIES FOR $p$ , $(p-s)$ AND MODIFIED $(p-s)$ -CONVEX FUNCTIONS

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*Abstract.* In this study, we develop three new quantum Hermite-Hadamard inequalities for the class of  $p$ ,  $(p-s)$  and modified type  $(p-s)$ -convex functions by utilizing left and right quantum integral. As special cases of these inequalities, we get known and new Hermite-Hadamard type inequality for modified type  $(p-s)$ -convex functions. The ideas and techniques of this article may be the starting point for further research in this field.

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