HERMITE–HADAMARD WEIGHTED INTEGRAL INEQUALITIES FOR (h,m)–CONVEX MODIFIED FUNCTIONS

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Abstract. In this paper, some new integral inequalities of the Hermite–Hadamard type are were obtained for (h,m)-convex modified functions. The results are obtained on the basis of the introduced definition of a generalized weighted integral operator by using the convexity property, the well-known Hölder's inequality and its modification. Some results existing in the literature are some special cases of our results.

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