

OPTIMAL (ω, c) -ASYMPTOTICALLY PERIODIC MILD SOLUTIONS TO SOME FRACTIONAL EVOLUTION EQUATIONS

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Abstract. In this article, we establish some new properties of the two-parameter Mittag-Leffler function and use them to prove that, mild solutions of the evolution equation ${}^C D_t^\alpha u(t) = Au(t) + f(t)$ ($t \in \mathbb{R}$) are (ω, c) -asymptotically periodic, where A is the generator of a strongly continuous semigroup $\{T(\theta)\}_{\theta \geq 0}$ (which is exponentially stable) on a Banach space \mathbb{X} and ${}^C D_t^\alpha$ denotes the Caputo fractional derivative of order $0 < \alpha \leq 1$. We further establish an existence and uniqueness result for optimal (ω, c) -asymptotically periodic mild solution if \mathbb{X} is a uniformly convex Banach space.

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