GENERALIZED FRACTIONAL OSTROWSKI TYPE INEQUALITIES VIA $h − s−$CONVEX FUNCTION

ALI HASSAN* AND ASIF RAZA KHAN

Abstract. We are introducing very first time a generalized class named it the class of $h − s$-convex functions. This class of functions contains many important classes including class of h-convex, Godunova-Levin s-convex, s-convex in the 2nd kind and hence contains class of convex and MT-convex functions. It also contains class of P-convex functions, class of Godunova-Levin functions and the class of ordinary convex functions. Also, we would like to state the generalization of the classical Ostrowski inequality via fractional integrals with respect to another function, which is obtained for functions whose first derivative in absolute values is $h − s$-convex function. Moreover we establish some Ostrowski type inequalities via fractional integrals with respect to another function and their particular cases for the class of functions whose absolute values at certain powers of derivatives are $h − s$-convex functions by using different techniques including Hölder's inequality and power mean inequality. Also, various established results would be captured as special cases. Moreover, some applications in terms of special means would also be given.

REFERENCES

[1] M. ALOMARI, M. DARUS, S. S. DRAGOMIR AND P. CERONE, *Ostrowski type inequalities for functions whose derivatives are s-convex in the second sense*, Appl. Math. Lett., **23**, (2010), 1071–1076.

[2] A. ARSHAD AND A. R. KHAN, *Hermite–Hadamard–Fejer Type Integral Inequality for $s − p$-Convex Functions of Several Kinds*, TJMM, **11** (2), (2019), 25–40.

[3] E. F. BECKENBACH, *Convex functions*, Bull. Amer. Math. Soc., **54**, (1948), 439–460.

[4] W. W. BRECKNER, *Stetigkeitsaussagen Fur Eine Klasse Verallgemeinerter Konvexer Funktionen in Topologischen Linearen Raumen*, (German), Publ. Inst. Math., **23** 37, (1978), 13–20.

[5] P. CERONE AND S. S. DRAGOMIR, *Ostrowski type inequalities for functions whose derivatives satisfy certain convexity assumptions*, Demonstratio Math, **37** (2), (2004), 299–308.

[6] W. CONGXIN AND M. MING, *On embedding problem of fuzzy number space: Part 1*, Fuzzy Sets and Systems, **44** (1), (1991), 33–38.

[7] W. CONGXIN AND G. ZENGTAI, *On Henstock integral of fuzzy-number-valued functions (I)*, Fuzzy Sets and Systems, **120** (1), (2001) 523–532.

[8] S. S. DRAGOMIR, *A Functional Generalization of Ostrowski Inequality via Montgomery identity*, Acta Math. Univ. Comenianae, LXXXIV, **1**, (2015), 63–78.

[9] S. S. DRAGOMIR, *Integral inequalities of Jensen type for λ-convex functions*, In Proceedings of RGMIA, Res. Rep. Coll., **1** (17), (2014).

[10] S. S. DRAGOMIR, *Inequalities of Jensen Type for $h − s$-Convex Functions*, Fasciculi Mathematici, **5**, (2015), 35–52.

[11] S. S. DRAGOMIR, *A Companion of Ostrowski's Inequality for Functions of Bounded Variation and Applications*, Int. J. Nonlinear Anal. Appl., **5** (2014), 89–97.

[12] S. S. DRAGOMIR, *On the Ostrowski's Integral Inequality for Mappings with Bounded Variation and Applications*, Math. Inequal. Appl., **4** (1), (2001), 59–66.

[13] S. S. DRAGOMIR, *Refinements of the Generalised Trapozoid and Ostrowski Inequalities for Functions of Bounded Variation*, Arch. Math., **91** (5), (2008), 450–460.

[14] S. S. DRAGOMIR AND N. S. BARNETT, *An Ostrowski Type Inequality for Mappings whose Second Derivatives are Bounded and Applications*, J. Indian Math. Soc. (N.S.), **66** (4) (1999), 237–245.

[15] S. S. DRAGOMIR, P. CERONE, N. S. BARNETT AND J. ROUMELIOTIS, *An Inequality of the Ostrowski Type for Double Integrals and Applications for Cubature Formulae*, Tamsui Oxf. J. Math. Sci., **16** (1), (2000), 1–16.

[16] S. S. DRAGOMIR, P. CERONE AND J. ROUMELIOTIS, *A new Generalization of Ostrowski Integral Inequality for Mappings whose Derivatives are Bounded and Applications in Numerical Integration and for Special Means*, Appl. Math. Lett., **13** (2000), 19–25.

[17] S. S.DRAGOMIR, J. PEČARIĆ AND L. PERSSON, *Some inequalities of Hadamard type*, Soochow J. Math., **21** (3), (1995), 335–341.

[18] A. EKINCI, *Klasik Eşitsizlikler Yoluyla Konveks Fonksiyonlar için Integral Eşitsizlikler*, Ph.D. Thesis, Thesis ID: 361162 in `tez2.yok.gov.tr`, Atatürk University, 2014.

[19] S. GAL, *Approximation theory in fuzzy setting*, Chapter 13 in Handbook of Analytic Computational Methods in Applied Mathematics (edited by G. Anastassiou), Chapman and Hall, CRC Press, Boca Raton, New York, (2000), 617–666.

[20] E. K. GODUNOVA, V. I. LEVIN, *Inequalities for functions of a broad class that contains convex, monotone and some other forms of functions*, Numerical Mathematics and Mathematical Physics, (Russian), **166**, (1985), 138–142.

[21] N. IRSHAD, A. R. KHAN AND A. NAZIR, *Extension of Ostrowki Type Inequality Via Moment Generating Function*, Adv. Inequal. Appl., **2**, (2020), 1–15.

[22] N. IRSHAD, A. R. KHAN AND M. A. SHAIKH, *Generalization of Weighted Ostrowski Inequality with Applications in Numerical Integration*, Adv. Ineq. Appl., **7**, (2019), 1–14.

[23] N. IRSHAD, A. R. KHAN, AND MUHAMMAD AWAIS SHAIKH, *Generalized Weighted Ostrowski-Gruss Type Inequality with Applications*, Global J. Pure Appl. Math., **15** (5), (2019), 675–692.

[24] N. IRSHAD AND A. R. KHAN, *On Weighted Ostrowski Gruss Inequality with Applications*, TJMM, **10** (1), (2018), 15–22.

[25] N. IRSHAD AND A. R. KHAN, *Generalization of Ostrowski Inequality for Differentiable functions and its applications to numerical quadrature rules*, J. Math. Anal., **8** (1), (2017), 79–102.

[26] O. KALEVA, *Fuzzy diferential equations*, Sets and Systems, **24** (1987) 301–317.

[27] M. MATŁOKA, *On Ostrowski type inequalities via fractional integrals of a function with respect to another function*, J. Nonlinear Sci. Appl., **13**, (2020), 100–106.

[28] D. S. MITRINOVIC, J. E. PECARIC AND A. M. FINK, *Inequalities Involving Functions and Their Integrals and Derivatives*, Kluwer Academic, Dordrecht, (1991).

[29] M. A. NOOR AND M. U. AWAN, *Some integral inequalities for two kinds of convexities via fractional integrals*, TJMM, **5**, (2013), 129–136.

[30] A. M. OSTROWSKI, *Uber die absolutabweichung einer differentiebaren funktion von ihrem integralmitelwert*, Comment. Math. Helv., **10**, (1938), 226–227.

[31] S. G. SAMKO, A. A. KILBAS, O. I. MARICHEV, *Fractional Integrals and Derivatives*, Theory and Applications Gordon and Breach New York, **1**, (1993).

[32] E. SET, *New inequalities of Ostrowski type for mappings whose derivatives are s-convex in the second sense via fractional integrals*, Comput. Math. Appl., **63**, (2012), 1147–1154.

[33] S. VAROŠANEC, *On h-convexity*, J. Math. Anal. Appl., **326** (1), (2007), 303–311.

[34] Z. G. XIAO, AND A. H. ZHANG, *Mixed power mean inequalities*, Research Communication on Inequalities, **8** (1), (2002), 15–17.

[35] X. YANG, *A note on Hölder inequality*, Appl. Math. Comput., **134**, (2003), 319–322.

Fractional Differential Calculus
www.ele-math.com
fdc@ele-math.com