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ON THE APPROXIMATE CONTROLLABILITY FOR FRACTIONAL NEUTRAL INCLUSION SYSTEMS WITH NONLOCAL CONDITIONS

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Abstract. The aim of this work is to study the approximate controllability for some fractional neutral inclusion system with nonlocal conditions. We establish a new variation of constant formula that helps us to formulate the problem of the approximate controllability. We assume that the linear system without the input functions is approximately controllable, then we prove with the lack of compactness, the approximate controllability for the whole nonlinear system. For illustrative purposes, we provide an application to the heat equation with memory.

Mathematics subject classification (2020): 93B05, 35R09, 35R11, 34G25, 34A08, 34Kxx, 34K09, 34K40.

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