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A STUDY OF A HIGH–ORDER TIME–FRACTIONAL PARTIAL DIFFERENTIAL EQUATION WITH PURELY INTEGRAL BOUNDARY CONDITIONS

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Abstract. The aim of this paper is to investigate the existence and uniqueness of the strong solution for the linear time-fractional partial differential equation with purely integral conditions. The aimed investigation is demonstrated based on the so-called energy inequality method and the density of the operator generated by the considered problem. To do so, we first set the position of the problem under consideration coupled with its corresponding equivalent problem, say problem (x). Afterward, we introduce some necessary functional spaces needed for exploring the existence and uniqueness of solution of problem (x). Finally, we investigate the existence and uniqueness of solution of the main operational equation.

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