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EXISTENCE AND STABILITY RESULTS FOR A PANTOGRAPH PROBLEM WITH SEQUENTIAL CAPUTO–HADAMARD DERIVATIVES

AMIRA ABDELNEBI* AND ZOUBIR DAHMANI

Abstract. In the current paper, we look at the existence, uniqueness, and stability of solutions for a new pantograph problem with three sequential derivatives of Caputo-Hadamard type. The proposed problem admits the third-order pantograph problem as a limiting case. So, based on Banach contraction principle and Leray-Schauder fixed point theorems, two main theorems are proved. Another main result for the Ulam-Hyers stability of solutions for the problem is established. Furthermore, an illustrative example is presented to show the applicability of the existence and uniqueness result as well as the Ulam stability one.

Mathematics subject classification (2020): 26A33, 34A08, 47H10.

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