

## EXISTENCE AND UNIQUENESS RESULTS FOR A CLASS OF FRACTIONAL DIFFERENTIAL EQUATIONS WITH NONLOCAL BOUNDARY CONDITIONS

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*Abstract.* This study focuses on constructing solutions for a specific type of second order fractional differential equation involving nonlocal boundary conditions. We additionally provide exemplifications that demonstrate the application of our results.

*Mathematics subject classification (2020):* 34A08, 34B10.

*Keywords and phrases:* Fractional differential equations, upper and lower solutions, monotone iterative technique, nonlocal boundary conditions.

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