

DETERMINATION OF INITIAL DATA IN TIME-FRACTIONAL WAVE EQUATION

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Abstract. In this paper, we consider a time-fractional wave equation for positive operators, including the classical Laplacian with the Dirichlet boundary condition. Determinations of initial velocity and perturbation are investigated. It is also shown that these inverse problems of determining the initial data are ill-posed. Moreover, under some conditions of well-posedness properties of the inverse problems are proved. As an appendix, we also provide some proof of the direct problems. Here, we develop the theoretical part of the inverse problems of finding the initial data for the time-fractional wave equations.

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