# INITIAL–BOUNDARY VALUE PROBLEMS TO THE
# TIME–SPACE NONLOCAL DIFFUSION EQUATION

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*Abstract.* This article investigates a time-fractional space-nonlocal diffusion equation in a bounded domain. The fractional operators are defined rigorously, using the Caputo fractional derivative of order $\beta$ and the Riemann-Liouville fractional integral of order $\alpha$, where $0 < \alpha < \beta \leqslant 1$. The solution is expressed as a series involving the two-parameter Mittag-Leffler function and orthonormal eigenfunctions of the Sturm-Liouville operator. The convergence of the series is investigated, and conditions for the solution to belong to a specific function space are established. The uniqueness of the solution is demonstrated and the continuity of the solution in the specified domain is confirmed through the uniform convergence of the series.

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