

QUASI-REVERSIBILITY METHOD FOR AN OPTIMAL CONTROL OF AN ILL-POSED FRACTIONAL DIFFUSION EQUATION

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Abstract. In this paper, an optimal control problem associated to an ill-posed fractional diffusion equation is considered. To study our initial problem, we use the quasi-reversibility method introduced by Lions and Lattès in 1969. More precisely, we consider an approximated optimal control problem of our initial problem. Then the new problem is associated to a well-posed state equation which approximate the ill-posed state equation. Firstly, we prove that the approximated optimal control problem admits a unique solution which we characterized using the Euler-Lagrange optimality conditions. Next, we show that the solution of the approximated optimal control problem converges to the solution of the initial optimal control problem. To finish, we characterize the optimal control of our initial problem by an optimality system.

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