DIRECT AND INVERSE PROBLEMS FOR A FRACTIONAL PARABOLIC EQUATION WITH MULTIPLE INVOLUTION

BATIRKHAN TURMETOV*, MAIRA KOSHANOVA,
MOLDIR MURATBEKOVA AND ISABEK ORAZOV

Abstract. In this paper, the concept of a nonlocal analogue of the Laplace operator is introduced. For the nonlocal parabolic equation with a fractional derivative in a cylindrical domain, the solvability of direct and inverse problems is studied. The problems are solved using the Fourier method. Theorems on the existence and uniqueness of solutions to the studied problems are proved.

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