

## STURM-LIOUVILLE AND BOUNDARY VALUE PROBLEMS IN NABLA DISCRETE FRACTIONAL CALCULUS

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*Abstract.* We consider a novel discrete nabla fractional self-adjoint operator  $L$  in the Caputo sense. We demonstrate basic properties of boundary-value problems under  $L$ , including explicit solutions. Furthermore, we consider Sturm-Liouville problems under  $L$  and prove eigenvalue results analogous to continuous classical Sturm-Liouville systems.

*Mathematics subject classification (2020):* 39A10, 39A70.

*Keywords and phrases:* Caputo fractional difference, Sturm-Liouville, fractional difference equation, nabla difference operator.

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