

## NEW FRACTIONAL INTEGRAL EXTENSIONS FOR INEQUALITIES INVOLVING MONOTONE FUNCTIONS

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**Abstract.** This paper extends classical results on integral inequalities involving monotone functions to the domain of Riemann-Liouville fractional integrals with positive arbitrary order  $\alpha$ . By employing a unified framework, our approach provides a more generalized understanding of the interplay between monotonicity and integrability in the case of fractional integration. We review classical results, introduce Riemann-Liouville integrals, and establish the fractional integral extensions. Our main results are presented, with discussions on their applications, contributing to a broader comprehension of this type of inequalities in mathematical analysis and its applications. *Mathematics subject classification* (2020): 26D15, 33B15.

**Keywords and phrases:** Integral inequality, monotone function, Riemann-Liouville integral.

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