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EXISTANCE OF NON–NEGATIVE AND NONDECREASING SOLUTION OF ERDÉLYI–KOBER FRACTIONAL INTEGRAL EQUATION WITH THE HELP OF SIMULATION TYPE CONDENSING OPERATOR

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Abstract. The purpose of this paper is to prove the existence of a solution for Erdélyi-Kober fractional integral equations using the generalised form of Darbo's fixed point theorem and a simulation form condensing operator in Banach space Ω. The primary purpose is to show that the Erdélyi-Kober fractional integral equation has solutions in $\mathbb{C}[0,1]$. Finally, to illustrate that our abstract conclusions are simple to verify, we give an example.

Mathematics subject classification (2020): 47H08, 47H09, 47H10, 34A12, 26A33.
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