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SOLVABILITY OF A FRACTIONAL DIFFERENTIAL PROBLEM FOR BEAM EQUILIBRIUM

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Abstract. In this work, we investigate a novel nonlinear differential problem involving Caputo derivatives. The problem features sequential derivatives that do not adhere to the semi-group and commutativity properties. Under certain specific conditions, the problem simplifies to a fourth-order ordinary problem, representing the static equilibrium of an elastic beam. Using the Banach contraction principle followed by Schaefer's fixed point theorem, we establish two key results regarding the uniqueness of solutions and the existence of at least one solution. We provide an example to validate one of these results. Additionally, we discuss Ulam-Hyers stability, including noteworthy limiting-case examples.

Mathematics subject classification (2020): 30C45, 39B72, 39B82.

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