Fractional Differential Calculus
Volume 14, Number 2 (2024), 295–317
doi:10.7153/fdc-2024-14-14

EMBEDDINGS IN RIEMANN–LIOUVILLE FRACTIONAL SOBOLEV SPACES AND APPLICATIONS

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Abstract. In this work, we present results on the embeddings of fractional Riemann-Liouville Sobolev spaces, using an important relationship between Riemann-Liouville Sobolev spaces and ordinary Sobolev spaces. This relationship allows us to prove compact embeddings after establishing continuous embeddings based on the continuity of the Riemann-Liouville fractional integral operators between Lebesgue spaces under certain conditions. We provide an example of a boundary problem where existence and uniqueness are addressed using two methods: the fixed point method and the Faedo-Galerkin method. Both methods require specific fractional type embeddings.

Mathematics subject classification (2020): 26A33, 34A08, 34B15.
Keywords and phrases: Riemann-Liouville fractional derivative, fractional Sobolev spaces, embedding, fixed point method, Faedo-Galerkin method.

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