OPERATORS OF FRACTIONAL CALCULUS AND ASSOCIATED INTEGRAL TRANSFORMS OF THE (p,q)–EXTENDED HURWITZ–LERCH ZETA FUNCTION

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Abstract. In this paper, our aim is to establish certain fractional integral and derivative formulas of the generalized (p,q)-extended Hurwitz-Lerch zeta function by using generalized Marichev-Saigo-Maeda fractional operators which involve, in their kernel, Appell's two-variable hypergeometric function $F_3(.)$. These results are expressed in terms of the Hadamard product (or the convolution) of two analytic functions in terms of (p,q)-extended Hurwitz-Lerch zeta function and Fox-Wright hypergeometric function ${}_r\Psi_s(.)$. We then obtain their composition formulas by using fractional integral and derivative formulas and certain Integral transforms associated with Beta, Laplace and Whittaker transforms involving generalized (p,q)-extended Hurwitz-Lerch Zeta function.

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