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CHAOTIC DYNAMICS AND CHAOS CONTROL IN A FRACTIONAL–ORDER 2D CHAOTIC MAP BASED ON THE CAPUTO DIFFERENCE OPERATOR

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Abstract. In the last few years, there has been significant interest in the literature of fractional calculus and its applications in nonlinear dynamical systems. This is especially true in fields such as physics, chemistry, biology, and engineering. This paper presents a fractional-order two-dimensional rational chaotic map. Using the Caputo-like delta difference operator, the fractional-order counterpart is constructed. By varying the system parameters and fractional order, the proposed fractional map can exhibit complex dynamic behavior. The chaotic dynamics are investigated by adopting several classical tools such as phase plots, bifurcation diagrams, the maximum Lyapunov exponent spectrum, and dynamical maps. In addition, the 0-1 test algorithm is presented to validate the chaotic behavior of the fractional map. In order to evaluate the complexity level of the fractional map, the C_0 algorithm and spectral entropy are employed. Finally, a nonlinear control law is designed to stabilize the state trajectories of the chaotic map towards zero. Computer simulations are carried out to illustrate and validate the theoretical results obtained in this paper.

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