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LYAPUNOV–TYPE INEQUALITIES FOR A RIEMANN–LIOUVILLE FRACTIONAL HYBRID BOUNDARY VALUE PROBLEMS

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Abstract. In this paper, we investigate a Riemann–Liouville fractional hybrid boundary value problem to delve into the complexities of fractional calculus. We introduce novel Lyapunov-type inequalities that are specifically adapted to the distinct features of the problem at hand. Our results enhance the theoretical framework and include a comprehensive example that highlights the practical significance and implications of our theoretical advancements.

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Paper FDC-15-06

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