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EXISTENCE AND APPROXIMATE CONTROLLABILITY FOR A CLASS OF FRACTIONAL ORDER HEMIVARIATIONAL INEQUALITIES

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Abstract. This paper explores the approximate controllability of a fractional differential control problem governed by a nonlinear hemivariational inequality in a Hilbert space. Initially, the existence of a mild solution for a fractional control inclusion problem, equivalent to the hemivariational inequality, is demonstrated using nonsmooth analysis and fixed-point techniques. Subsequently, sufficient conditions for the approximate controllability of the inclusion problem are established, assuming that the corresponding linear system is approximately controllable. The existence and controllability results derived for the inclusion problem are applicable to the nonlinear hemivariational problem under consideration. An example is presented to illustrate the effectiveness of the proposed results.

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