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THEORETICAL ANALYSIS OF A CLASS OF NONLOCAL ϕ–CAPUTO FRACTIONAL NONLINEAR EVOLUTION EQUATIONS USING THE MEASURE OF NON–COMPACTNESS IN BANACH SPACES

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Abstract. This study explores the existence of solutions for nonlocal fractional differential evolution equations through the concept of the measure of non-compactness. Our approach incorporates probability density functions, operator semigroup theory, and the Mönch fixed point theorem. To demonstrate the practical significance of our findings, we conclude with an application.

Mathematics subject classification (2020): 26A33, 34K37, 34A08.
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Paper FDC-15-08

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