Fractional
Differential
Calculus
Volume 15, Number 2 (2025), 221–231 doi:10.7153/fdc-2025-15-13

CONTROL OF SYSTEMS OF LINEAR DIFFERENTIAL EQUATIONS IN THE RIEMANN–LIOUVILLE SENSE

JOSÉ VILLA-MORALES

Abstract. In this paper, we prove a bang-bang principle and a maximum principle, analogous to Pontryagin's, for systems of linear fractional differential equations in the Riemann-Liouville sense. We then apply these results to two examples, illustrating how incorporating fractional dynamics can improve optimal arrival times.

Mathematics subject classification (2020): 26A33, 93D15, 93C15, 93C05.
Keywords and phrases: Bang-bang principle, Pontryagin principle, Riemann-Liouville differential equations, controllability.

REFERENCES

[1] S. H. ABEL-GAID, A. H. QAMLO, B. G. MOHAMED, *Bang-Bang Property and Time-Optimal Control for Caputo Fractional Differential Systems*, Fractal and Fractional **8**, 2 (2024), 84.
[2] G. M. BAHAA, *Fractional optimal control problem for differential system with delay argument*, Advances in difference equations **2017**, (2017), 1–19.
[3] H. BREZIS, *Functional analysis, Sobolev spaces and partial differential equations*, Springer, 2011.
[4] L. C. EVANS, *An introduction to mathematical optimal control theory version 0.2*, Lecture notes available at http://math.berkeley.edu/evans/control.course.pdf, 1983.
[5] W. H. FLEMING, R. W. RISHEL, *Deterministic and stochastic optimal control*, Springer Science & Business Media **1**, 2012.
[6] G. B. FOLLAND, *Real analysis: modern techniques and their applications*, John Wiley & Sons **40**, 1999.
[7] L. M. HOCKING, *Optimal control: an introduction to the theory with applications*, Oxford University Press 1991.
[8] A. A. KILBAS, H. M. SRIVASTAVA, J. J. TRUJILLO, *Theory and applications of fractional differential equations*, Elsevier **204**, 2006.
[9] A. A. KILBAS, O. I. MARICHEV, G. SAMKO, *Fractional integrals and derivatives : theory and applications*, Gordon and Breach Science Publishers, 1993.
[10] T. LIAN, Z. FAN, G. LI, *Time optimal controls for fractional differential systems with Riemann-Liouville derivatives*, Fractional Calculus and Applied Analysis **21**, 6 (2018), 1524–1541.
[11] D. G. LUENBERGER, Y. YE, *Linear and nonlinear programming*, Addison-wesley **2**, 1984.
[12] Q. LÜ, *Bang-bang principle of time optimal controls and null controllability of fractional order parabolic equations*, Mathematica Sinica, English Series **26**, 12 (2010), 2377–2386.
[13] J. MACKI, A. STRAUSS, *Introduction to optimal control theory*, Springer Science, 2012.
[14] I. MATYCHYN, V. ONYSHCHENKO, *Time–Optimal Control of Linear Fractional Systems with Variable Coefficients*, International Journal of Applied Mathematics and Computer Science **31**, 3 (2021), 375–386.
[15] X. PAN, X. LI, J. ZHAO, *Solvability and Optimal Controls of Semilinear Riemann-Liouville Fractional Differential Equations*, Abstract and Applied Analysis **2014**, 1 (2014), 216919.
[16] D. QIAN, C. LI, R. P. AGARWAL, P. J. WONG *Stability analysis of fractional differential system with Riemann-Liouville derivative*, Mathematical and Computer Modelling **52**, 5-6 (2010), 862–874.
[17] M. REED, B. SIMON, *I: Functional analysis*, Academic press, 1981.

Paper FDC-15-13

[18] S. REZAPOUR, S. K. NTOUYAS, A. AMARA, S. ETEMAD, J. TARIBOON, *Some existence and dependence criteria of solutions to a fractional integro-differential boundary value problem via the generalized Gronwall inequality*, Mathematics **9**, 11 (2021), 1165.

[19] N. H. SWEILAM, S.M. AL-MEKHLAFI, T. ASSIRI, A. ATANGANA, *Optimal control for cancer treatment mathematical model using Atangana-Baleanu-Caputo fractional derivative*, Advances in Difference Equations **2020**, 1 (2020), 334.

[20] J. VILLA-MORALES, L. RINCÓN, G. BECERRA-GUZMÁN, *Optimal controls for linear fractional differential equations in the Caputo sense*, Journal of Computational and Applied Mathematics **470**, (2025).

[21] J. ZABCZYK, *Mathematical control theory*, Springer, 2020.

Fractional Differential Calculus
www.ele-math.com
fdc@ele-math.com