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ON CAPUTO TEMPERED FRACTIONAL COUPLED SYSTEMS WITH THREE POINTS BOUNDARY CONDITIONS

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Abstract. In this paper, we investigate the existence, uniqueness, and Ulam stability of solutions to systems of Caputo-tempered fractional differential equations subject to three-point boundary conditions. The analysis is grounded in an extended version of Perov's fixed point theorem and a Krasnoselskii-type approach. By integrating these methods with techniques involving vector-valued metrics and matrix sequences that converge to zero, we establish their primary theoretical results. To further elucidate the findings, illustrative examples are provided in the final section of the paper.

Mathematics subject classification (2020): 26A33, 34A08, 34K37.
Keywords and phrases: Three point boundary value problems, Caputo tempered fractional derivative, Perov fixed point theorem, existence, uniqueness, Ulam stability, vector-valued norm, convergent to zero matrices.

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