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ANALYSIS OF A CLASS OF Δ–FRACTIONAL HYBRID DIFFERENTIAL EQUATIONS INVOLVING THE ϕ–CAPUTO DERIVATIVE ON TIME SCALES

NAJAT CHEFNAJ*, ABDELLAH TAQBIBT, KHALID HILAL AND M'HAMED ELOMARI

Abstract. This paper explores the existence of solutions for fractional hybrid differential equations that involve the ϕ-Caputo derivative, defined on time scales. The ϕ-Caputo derivative extends the classical Caputo derivative by adapting it to time scales, making it possible to model systems that exhibit both continuous and discrete behavior. This unique characteristic allows the ϕ-Caputo derivative to capture dynamics that occur across varying time intervals, providing a more versatile framework for mathematical modeling.

To demonstrate the existence of solutions, the study leverages Dhage's fixed point theorem is a powerful and widely recognized tool for establishing the existence of fixed points in Banach algebras. By applying this theorem to fractional hybrid differential equations on time scales, the research introduces an innovative approach that bridges theoretical concepts with practical application.

To validate the correctness and real-world relevance of the theoretical findings, the study includes a practical example involving uncertainty modeling in physical systems. This example illustrates how the proposed method can be applied in scenarios where both deterministic and uncertain behaviors are present.

The outcomes of this research offer promising potential for various fields, including biology, engineering, and control theory, where dynamic systems often require flexible mathematical frameworks to address complex behavior.

Mathematics subject classification (2020): 26A33, 34K37, 34A08.
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REFERENCES

[1] A. AHMADKHANLU AND M. JAHANSHAHI, *On the existence and uniqueness of solution of initial value problem for fractional order differential equations on time scales*, Bull. Iranian Math. Soc. **38**, 1 (2012), 241–252.

[2] S. AHMED, A. AHMED, I. MANSOOR, F. JUNEJO, A. SAEED, *Output feedback adaptive fractional-order super-twisting sliding mode control of robotic manipulator*, Iran. J. Sci. Technol. Trans. Elect. Eng. **45**, (2021), 335–347.

[3] S. Í. ARAZ, *Analysis of a Covid-19 model: optimal control, stability, and simulations*, Alexandria Eng. J. **60**, 1 (2021), 647–658.

[4] A. ATANGANA, S. ÍĞRET ARAZ, *Mathematical model of COVID-19 spread in Turkey and South Africa: theory, methods, and applications*, Adv. Dif. Equ. **2020**, 1(2020), 1–89.

[5] M. AWADALLA, Y. YAMENI, *Modeling exponential growth and exponential decay real phenomena by ϕ-Caputo fractional derivative*, J. Adv. Mathe. Comput. Sci. **28**, 2 (2018), 1–13.

[6] A. CERNEA, *On a fractional differential inclusion with Maxima*, Frac. Calc. Appl. Anal. **19**, 5 (2016), 1292–1305.

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[7] N. CHEFNAJ, K. HILAL AND A. KAJOUNI, *Existence and uniqueness of solutions for ϕ-Caputo fractional neutral sequential differential equations on time scales*, Journal of Applied Mathematics and Computing **70**, 6 (2024), 5251–5268.

[8] N. CHEFNAJ, K. HILAL AND A. KAJOUNI, *Impulsive ϕ-Caputo hybrid fractional differential equations with non-local conditions*, J. Math. Sci. (2023), 1–12.

[9] N. CHEFNAJ, K. HILAL AND A. KAJOUNI, *The existence, uniqueness and Ulam-Hyers stability results of a hybrid coupled system with ϕ-Caputo fractional derivatives*, Journal of Applied Mathematics and Computing **70**, 3 (2024), 2209–2224.

[10] N. CHEFNAJ, A. TAQBIBT, K. HILAL AND S. MELLIANI, *Study of nonlocal boundary value problems for hybrid differential equations involving ϕ-Caputo Fractional Derivative with measures of noncompactness*, J. Math. Sci. (2023), 1–10.

[11] B. C. DHAGE, *On a fixed point theorem in Banach algebras with applications*, Appl. Math. Lett. 18 (2005) 273-280.

[12] E. KARAPINAR, E. BENKHETTOUD, J. E. LAZREGD AND M. BENCHOHRAD, *Fractional differential equations with maxima on time scale via Picard operators*, Filomat **37**, 2 (2023), 393–402.

[13] D. OTROCOL, *Hybrid differential equations with maxima via Picard operators theory*, Stud. Univ. Babes. Bolyai. Math. **61**, (2016), 421–428.

[14] A. TAQBIBT, N. CHEFNAJ, K. HILAL, S. MELLIANI, *ϕ-Caputo fractional differential equations with maxima on time scales*, Journal of Mathematical Sciences, (2024), 1–13.

[15] A. TAQBIBT, M. ELOMARI AND S. MELLIANI, *Nonlocal semilinear ϕ-Caputo fractional evolution equation with a measure of noncompactness in Banach space*, Filomat **37**, 20 (2023), 6877–6890.