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MULTIPLICITY OF SOLUTIONS FOR HOMOGENEOUS FRACTIONAL HAMILTONIAN SYSTEMS

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Abstract. This paper investigates the multiplicity of solutions for a class of fractional Hamiltonian systems defined by the system:

$$\begin{cases} {}_tD^{\alpha}_{\infty}({}_{-\infty}D^{\alpha}_t u)(t) + L(t)u(t) = -a(t)\nabla G(u(t)) + b(t)\nabla H(u(t)) + h(t), & t \in \mathbb{R} \\ u \in H^{\alpha}(\mathbb{R}), \end{cases}$$

where ${}_tD^{\alpha}_{\infty}$ and ${}_{-\infty}D^{\alpha}_t$ denote the Liouville-Weyl fractional derivatives with $\frac{1}{2} < \alpha < 1$, $L(t)$ is a symmetric and positive definite matrix in $\mathbb{R}^{N\times N}$, $a(t)$ and $b(t)$ are positive bounded functions, $G(u)$ and $H(u)$ are homogeneous functions on $\mathbb{R}^N$, and $h(t)$ is a given function in $\mathbb{R}^N$. Using variational techniques and the Pohozaev fibering method, we establish the existence of infinitely many solutions when $h(t) = 0$, and at least three solutions when $h(t)$ is non-trivial but sufficiently small. These results are novel and extend previous findings in the literature.

Mathematics subject classification (2020): 34C37, 35A15, 35B38.
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