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LINKING AND EXISTENCE RESULT FOR THE FRACTIONAL p–LAPLACIAN PROBLEMS INVOLVING SINGULAR NONLINEARITY

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Abstract. The purpose of the work is to investigate whether solutions exist for a certain class of fractional non-linear equations that are non-local and feature both singular and subcritical nonlinearities. The equation is given as follow

$$(\mathscr{P}_\lambda)\quad \begin{cases} (-\Delta_p)^s u = \lambda |u|^{p-2}u + \dfrac{\eta}{u^\delta} + \beta(x)|u|^{q-2}u & \text{in } \Omega, \\ u > 0 & \text{in } \Omega, \\ u = 0 & \text{in } \mathbb{R}^N \setminus \Omega. \end{cases}$$

Here Ω is a bounded domain of $\mathbb{R}^N$ with $N \geqslant 2$ and Lipschitz boundary $\partial\Omega$, $\lambda, \eta > 0$ are two a real parameters, $(-\Delta_p)^s$ represents the fractional p-Laplacian operator with $s \in (0,1)$ and $p > 1$ satisfies $sp < N$, $q \in (p, p_s^\star)$. $\beta : \Omega \to \mathbb{R}$ is a bounded function, δ is a positive real number, satisfying $\delta \in (0,1)$. The study makes use of variational methods to prove that solutions exist. The author uses some abstract linking theorem based on the $\mathscr{Z}_2$-cohomological index to determine the critical points of a suitable functional that is related to the equation. The paper shows that the equation has at least one nontrivial solution for any positive value of the parameter λ.

Mathematics subject classification (2020): 35D30, 35J60, 35J75, 35R11, 46E35.
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