

## A DILUTION TEST FOR THE CONVERGENCE OF SUBSERIES OF A MONOTONE SERIES

LASSE LESKELÄ AND MIKKO STENLUND

*Abstract.* Cauchy's condensation test allows to determine the convergence of a monotone series by looking at a weighted subseries that only involves terms of the original series indexed by the powers of two. It is natural to ask whether the converse is also true: Is it possible to determine the convergence of an arbitrary subseries of a monotone series by looking at a suitably weighted version of the original series? In this note we show that the answer is affirmative and introduce a new convergence test particularly designed for this purpose.

*Mathematics subject classification (2010):* 40A05.

*Keywords and phrases:* condensation test, dilution test, subseries, thinning, polynomially sparse subsequence, counting function.

### REFERENCES

- [1] C. R. BANERJEE AND B. K. LAHIRI, *On subseries of divergent series*, Amer. Math. Monthly, 71:767–768, 1964.
- [2] G. H. BEHFOROZ, *Thinning out the harmonic series*, Math. Mag., 68(4):289–293, 1995.
- [3] D. D. BONAR AND M. KHOURY, JR., *Real Infinite Series*, Mathematical Association of America, Washington DC, 2006.
- [4] G. H. HARDY AND E. M. WRIGHT, *An Introduction to the Theory of Numbers*, Oxford University Press, sixth edition, 2008.
- [5] A. J. KEMPER, *A curious convergent series*, Amer. Math. Monthly, 21(2):48–50, 1914.
- [6] T. SCHMELZER AND R. BAILLIE, *Summing a curious, slowly convergent series*, Amer. Math. Monthly, 115(6):525–540, 2008.