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INTEGRAL REPRESENTATION OF SCHLÖMILCH SERIES

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Abstract. Certain integral representations are derived for the Schlömilch series of Bessel functions of the first kind J_ν, using newly derived integral representation of first kind Kapteyn–type series.

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