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ON CERTAIN SERIES INVOLVING RECIPROCALS OF BINOMIAL COEFFICIENTS

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Abstract. We evaluate the following family of series involving reciprocals of binomial coefficients in terms of elementary functions for $m = 3, 4$.

$$\sum_{k=0}^{\infty} \frac{x^k}{(mk+1)\binom{mk}{k}}.$$

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