CERTAIN SUBCLASSES OF BI–UNIVALENT
FUNCTIONS SATISFYING SUBORDINATE CONDITIONS

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Abstract. In this paper, we introduce and investigate each of the following subclasses:

$$\mathscr{S}_\Sigma(\lambda,\gamma;\varphi), \quad \mathscr{HS}_\Sigma(\alpha), \quad \mathscr{R}_\Sigma(\eta,\gamma;\varphi) \quad \text{and} \quad \mathscr{B}_\Sigma(\mu;\varphi)$$

$$(0 \leqslant \lambda \leqslant 1; \ \gamma \in \mathbb{C} \setminus \{0\}; \ \alpha \in \mathbb{C}; \ 0 \leqslant \eta < 1; \ \mu \geqslant 0)$$

of bi-univalent functions, φ is an analytic function with positive real part in the unit disk $\mathbb{D}$, satisfying $\varphi(0) = 1$, $\varphi'(0) > 0$, and $\varphi(\mathbb{D})$ is symmetric with respect to the real axis. We obtain coefficient bounds involving the Taylor-Maclaurin coefficients $|a_2|$ and $|a_3|$ of the function f when f is in these classes. The various results, which are presented in this paper, would generalize and improve those in related works of several earlier authors.

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