ASYMPTOTIC FORMULAE ASSOCIATED WITH THE WALLIS POWER FUNCTION AND DIGAMMA FUNCTION

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Abstract. Let s,t be two given real numbers, $s \neq t$. We determine the coefficients $c_j(s,t)$ such that

$$\left[\frac{\Gamma(x+t)}{\Gamma(x+s)} \right]^{1/(t-s)} \sim \exp\left(\psi\left(x + \sum_{j=0}^{\infty} c_j(s,t)x^{-j} \right) \right)$$

as $x \to \infty$, where $\psi(x) = \Gamma'(x)/\Gamma(x)$ denotes the digamma function. Also, the analysis of the coefficients in the asymptotic expansion of the composition $\exp(\psi(x+s))$ is given in details.

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